Self-Enforcing Debt, Reputation, and the Role of Interest Rates

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The Motivation

- Why does a sovereign country repay its debt?
- Why does an investor lend to a sovereign country?

A First Answer

Reputational Argument

- Sovereign states who repudiate their debts may tarnish their reputation
- They may be denied access to financial markets in future periods
- And loose their ability to smooth consumption and share risks

Eaton, J. and Gersovitz, M.
Debt with Potential Repudiation: Theoretical and Empirical Analysis
Review of Economic Studies (1981)

A Second Answer

Bulow, J. and Rogoff, K.
Sovereign Debt: Is to Forgive or Forget?
American Economic Review (1989)

- They studied repayment incentives of a small open economy borrowing from competitive, risk-neutral investors
- The default punishment is exclusion from future borrowing
  - After default, a sovereign can save (purchase the debt issued by other countries)
- Assumption on interest rates and debt limits:
  - Present value of future endowments is finite: *interest rates are higher than growth rates*
  - Debt limits are tighter than the natural debt limits

A Second Answer

Definition (Bulow and Rogoff)

Reputation debt is the amount of debt sustained exclusively by the threat of credit exclusion

Theorem (Reputation Debt Cannot Be Sustained)

1. *The sanction alone of refusal of future loans cannot support positive levels of debt*
2. *If there are domestic costs (by means of output drop) in addition to exclusion from credit markets, then debt can be sustained, but only on the basis of these costs*

> *"Even if some lending is feasible because of direct sanctions, having a reputation for repayment in no way enhances the ability to borrow"*

Subsequent Literature

Vast literature proposing alternative mechanisms to answer why countries repay their debt in the absence of sanctions

- Cole and Kehoe (JMonE 1995)
- Cole and Kehoe (IER 1998)
- Kletzer and Wright (AER 2000)
- Kehoe and Perri (ECMA 2002)
- Gul and Pesendorfer (ECMA 2004)
- Krueger and Uhlig (JMonE 2006)
- Amador (2012)

A GE Version of Bulow and Rogoff

Hellwig, C. and Lorenzoni, G.
Bubbles and self-enforcing debt
Econometrica (2009)

- Consider a general equilibrium version of BR
- Show, by means of an example, that debt can be sustained

What is the GE effect? Role of Interest Rates

- In BR, interest rates are exogenous and assumed to be higher than growth rates
- In HL, interest rates are endogenous and may adjust downward, providing repayment incentives

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Intuition for the Role of Interest Rates

- The utility after default is endogenous and depends on market interest rates
  - The defaulting country can save to smooth consumption
- Lower interest rates make both borrowing more appealing and saving after default less appealing
- HL provide an example where interest rates are low enough (lower than endowment growth rates) to sustain positive debt limits through a bubble mechanism
  - Debt limits satisfy exact roll-over: maximum level of outstanding debt can be exactly refinanced by issuing new claims

Our Contribution

Issue

- How domestic costs of default do interact with the threat of credit exclusion to determine interest rates and sustainable debt?
- Analyze reputation debt when there is a drop in output after default

Results

Positive levels of self-enforcing debt

Whatever small is the output drop, equilibrium interest rates are always higher than growth rates of any country

Debt levels must be bounded by natural debt limits

In particular, we cannot have a bubble component in debt limits

Interpretation of our Results

There is debt, but ...

- Is it due only to direct sanctions (output drop debt)?
- Is part of the debt due to the threat of credit exclusion (reputation debt)?

Another Contribution

- We construct an example showing that a country's reputation debt can be sustained even if interest rates are higher than its growth rates
- In a general equilibrium environment, a country can sustain reputation debt through a bubble even if its repayment incentives are the same as in Bulow and Rogoff
- We highlight financial intermediation as a possible channel for creditworthiness
- If reputation debt is sustained, interest rates must be lower than someone's (not necessarily the borrowers) growth rates

Role of Interest Rates ...

Not only for repayment incentives, but also related to lending incentives

Outline

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1. The Model
   - The Physical Environment
   - Markets and Equilibrium
   - Literature: BR and HL

2. Sustaining Reputation Debt without Bubbles ($\lambda > 0$)
   - Interest Rates Must be Higher than Growth Rates
   - Asymptotic Properties When $\lambda \to 0$
   - Disentangling Repayment Incentives

3. Repaying More than the Natural Ability to Repay ($\lambda = 0$)
   - Intermediation as a Reputation Mechanism
   - Indeterminacy of Debt Limits with Real Effects
   - The Need for Low Interest Rates

The Physical Environment

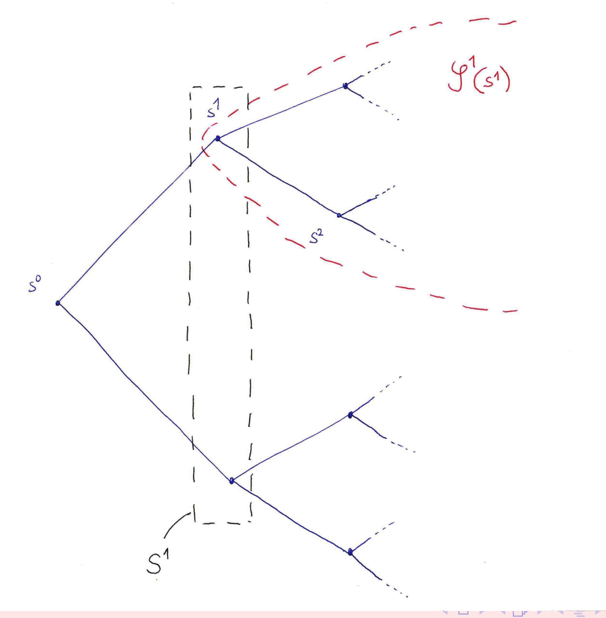
One-good, stochastic, infinite horizon endowment economy

- a finite set I of agents (countries) whose income is subject to random shocks
- An event tree $\mathcal{S}$ with an initial date-0 event $s^0 \in \mathcal{S}$
- For $t \in \{1, 2, ...\}$ there is a finite set $S^t \subset \mathcal{S}$ of date-t events s^t
- Each $s^t \in S^t$ has a unique predecessor in S^{t-1} and a finite number of successors $s^{t+1} \succ s^t$ in S^{t+1}
- For any $\tau > t$, we use the notation

$$s^\tau \succ s^t$$

when the date τ event s^τ belongs to the subtree starting at s^t

Event-Tree



Agent i

- Income process

$$y^i = (y^i(s^t))_{s^t \in \mathcal{S}}$$

with $y^i(s^t) > 0$

- Intertemporal preferences

$$U(c) = \sum_{t \geqslant 0} \beta^t \sum_{s^t \in S^t} \pi(s^t) u(c(s^t))$$

- Continuation utility at event s^t is denoted

$$U(c|s^t) := u(c(s^t)) + \sum_{\tau \geqslant 1} \beta^\tau \sum_{s^{t+\tau} \succ s^\tau} \pi(s^{t+\tau}|s^t) u(c(s^{t+\tau}))$$

Sequential Trading of One Period Contingent Bonds

- The country has access to a complete set of one period contingent bonds (Arrow securities)
- Denote by $a^i(s^{t+1})$ country i's bond-holding (chosen at event s^t) of the security which delivers at the successor event $s^{t+1} \succ s^t$
- Since trade occurs sequentially, one should impose debt limits (to avoid Ponzi schemes)
- Denote by $D^i(s^{t+1})$ country's i debt limit on the security paying at s^{t+1}
- $q(s^{t+1})$ denotes price (in units of s^t-consumption) of the security paying contingent to event s^{t+1}
- Denote by $a^i(s^0)$ agent i's initial financial claim (un-modeled past)

Budget Set $B^i(D^i, a^i(s^0)|s^0)$

The country has access to a complete set of one period contingent bonds

- solvency at s^0

$$c^i(s^0) + \sum_{s^1 \succ s^0} q(s^1)a^i(s^1) \leqslant y^i(s^0) + \underbrace{a^i(s^0)}_{primitive}$$

- solvency at every $s^t \succ s^0$

$$c^i(s^t) + \sum_{s^{t+1} \succ s^t} q(s^{t+1})a^i(s^{t+1}) \leqslant y^i(s^t) + a^i(s^t)$$

- debt constraints

$$a^i(s^{t+1}) \geqslant -D^i(s^{t+1})$$

Default Option

- In the above definition of a sequential equilibrium agents are not given the option to default
- At any event s^t the country can refuse to honour its contract and default on its promises

Consequences of Default

1. Exclusion from credit markets: No borrowing after default
2. Output drop
3. Saving is possible after default

Output Drop

- It has been documented that output falls during sovereign default
  - Cohen (NBER 1992), Tomz and Wright (JEEA 2007) and Mendoza and Yue (QJE 2012)
  - Disruption of international trade (imported inputs) or domestic financial systems (financing cost of working capital)
- We model output drop as an exogenous fraction $\lambda \in (0, 1)$ of endowment
  - Bulow and Rogoff (AER 1989)
  - Cole and Kehoe (RES 2000)
  - Aguiar and Gopinath (JIE 2006)
  - Arellano (AER 2008)
  - Bai and Zhang (ECMA 2010, JIE 2012)
  - Ábrahám and Cárceles-Poveda (JET 2010)

Self-Enforcing Debt Constraints

- Markets are complete: lenders have no incentives to provide credit contingent to some events if they anticipate that the borrower will default
- There is no partial default since the default punishment is independent of the default level
- The bounds should be compatible with repayment incentives: self-enforcing debt limits

Alvarez, F. and Jermann, U. J.
Efficiency, Equilibrium, and Asset Pricing with Risk of Default
Econometrica (2000)

No-Default Trade Opportunities at event s^τ

Let $B^i(D^i, b|s^\tau)$ be the set of all (c, a) satisfying

- solvency at s^τ

$$c^i(s^\tau) + \sum_{s^{\tau+1} \succ s^\tau} q(s^{\tau+1})a^i(s^{\tau+1}) \leqslant y^i(s^\tau) + b$$

- solvency at every $s^t \succ s^\tau$

$$c^i(s^t) + \sum_{s^{t+1} \succ s^t} q(s^{t+1})a^i(s^{t+1}) \leqslant y^i(s^t) + a^i(s^t)$$

- debt constraints at every $s^t \succ s^\tau$: $a^i(s^t) \geqslant -D^i(s^t)$

Continuation utility

$$J^i(D^i, b|s^\tau) := \sup\{U(c^i|s^\tau) \; : \; (c^i, a^i) \in B^i(D^i, b|s^\tau)\}$$

Trade Opportunities at event s^τ after Default

Let $B^i_\lambda(0,0|s^\tau)$ be the set of all (c,a) satisfying

- solvency at s^τ

$$c^i(s^\tau) + \sum_{s^{\tau+1} \succ s^\tau} q(s^{\tau+1}) a^i(s^{\tau+1}) \leqslant (1-\lambda) y^i(s^\tau) + 0$$

- solvency at every $s^t \succ s^\tau$

$$c^i(s^t) + \sum_{s^{t+1} \succ s^t} q(s^{t+1}) a^i(s^{t+1}) \leqslant (1-\lambda) y^i(s^t) + a^i(s^t)$$

- no borrowing at every $s^t \succ s^\tau$: $a^i(s^t) \geqslant 0$

Outside option

$$J^i_\lambda(0,0|s^\tau) := \sup\{U(c^i|s^\tau) \ : \ (c^i,a^i) \in B^i_\lambda(0,0|s^\tau)\}$$

Self-Enforcing Debt Constraints

- The function $b \mapsto J^i(D, b|s^t)$ is strictly increasing

Definition

The constraints D^i_λ are self-enforcing and not-too-tight (ntt) if

$$\forall s^t \succ s^0, \quad J^i(D^i_\lambda, -D^i_\lambda(s^t)|s^t) = J^i_\lambda(0, 0|s^t)$$

- Investors have no incentives to lend more that $D^i_\lambda(s^t)$
- If $a^i(s^t) < -D^i_\lambda(s^t)$, then agent i prefers default to debt repayment

Remark

Not-too-tight debt limits are determined at equilibrium and taken as given by agents, like prices

Competitive Equilibrium with Self-Enforcing Debt

A competitive equilibrium with self-enforcing debt is a family

$$(q, (c^i, a^i, D^i_\lambda)_{i \in I})$$

with

- optimal individual choices

$$(c^i, a^i) \in d^i(D^i_\lambda, a^i(s^0)|s^0)$$

- markets clear

$$\sum_{i \in I} c^i = \sum_{i \in I} y^i \quad \text{and} \quad \sum_{i \in I} a^i = 0$$

- debt limits are not-too-tight,

$$J^i(D^i_\lambda, -D^i_\lambda(s^t)|s^t) = J^i_\lambda(0, 0|s^t)$$

Present Values

- The price in units of date-0 consumption of a contract paying one unit of consumption in the event s^t is denoted $p(s^t)$

$$p(s^0) = 1 \quad \text{and} \quad p(s^{t+1}) = q(s^{t+1})p(s^t)$$

- Present value at event s^t

$$\mathrm{PV}(x|s^t) = \frac{1}{p(s^t)} \sum_{s^\tau \succeq s^t} p(s^\tau)x(s^\tau)$$

Level of Interest Rates

Definition

Interest rates are higher than agent i's growth rates when $\mathrm{PV}(y^i|s^0) < \infty$, and lower when $\mathrm{PV}(y^i|s^0) = \infty$

Interpretation

- Assume that

$$q(s^{t+1}) = \frac{\pi(s^{t+1}|s^t)}{(1+r)} \quad \text{and} \quad y(s^{t+1}) = (1+g)y(s^t)$$

- Interest rates are higher than growth rates if, and only if, $r > g$
- Interest rates are lower than growth rates if, and only if, $r \leqslant g$

Natural Ability to Borrow

- Assume that interest rates are higher than agent i's growth rates
- Agent i's natural ability to repay at event s is given by

$$\mathrm{PV}(y^i|s^t)$$

- We call this amount the natural debt limit, denoted by $N^i(s^t)$

First BR Theorem

Theorem

Assume that there is no output drop ($\lambda = 0$), and interest rates are higher than agent i's growth rates. If D_0^i is not-too-tight and tighter than natural debt limits, then $D_0^i = 0$

- There is no reputation debt

▸ Proof

Second BR Theorem

Theorem

Assume that there is output drop ($\lambda > 0$), and interest rates are higher than agent i's growth rates. If D^i_λ is not-too-tight and tighter than natural debt limits, then $D^i_\lambda = \mathrm{PV}(\lambda y^i | s^t)$

- Debt can be sustained, but only on the basis of the output drop
- There is no reputation debt

HL Theorem

Theorem

Assume that there is no output drop ($\lambda = 0$). If D_0^i is not-too-tight, then they form a bubble in the sense that they allow for exact roll-over

$$\forall s^t \in \mathcal{S}, \quad D_0^i(s^t) = \sum_{s^{t+1} \succ s^t} q(s^{t+1}) D_0^i(s^{t+1})$$

- Outstanding debt can be exactly refinanced by issuing new claims
- No ad-hoc assumptions on endogenous variables
- Application: reputation debt can be sustained

Hellwig and Lorenzoni (2009)

- They consider a simple (symmetric) economy with two agents, two shocks (high and low)
- They prove the existence of a symmetric Markov equilibrium
- Positive levels of debt are sustained because interest rates are lower than every agent's growth rates (zero risk-free interest rate)
- Interest rates are low enough ($\mathrm{PV}(y^i|s^0) = \infty$) to provide repayment incentives

Partial to General Equilibrium

The First BR Theorem does not extend to general equilibrium

Outline

Extending the Literature

- The Second BR Theorem imposes an ad-hoc assumption on interest rates
- The HL Theorem focusses on the case $\lambda = 0$
- We propose to extend these two results:
  - Allowing output costs after default: $\lambda > 0$
  - Without assuming that interest rates are higher than growth rates

Output Drop and Sustainable Debt

Theorem

Assume there is output drop after default ($\lambda > 0$). If D^i_λ is not-too-tight, then

- *interest rates must be higher than growth rates* ▸ Proof
- *there exists a non-negative M^i allowing for exact roll-over such that*

$$D^i_\lambda(s^t) = \mathrm{PV}(\lambda y^i | s^t) + M^i(s^t)$$

▸ Proof

- BR assumed a priori that interest rates are higher than growth rates
- There is no need to make this assumption: this is a necessary condition

Self-Enforcing Debt at Equilibrium

Corollary

Assume there is output drop after default ($\lambda > 0$). At any competitive equilibrium with self-enforcing debt, interest rates are higher than any agent's growth rates and

$$D^i_\lambda(s^t) = \mathrm{PV}(\lambda y^i | s^t)$$

- Bubbles are not compatible with market clearing and high interest rates
- The proof is based on the following market transversality result

$$\lim_{t\to\infty} \sum_{s^t \in S^t} p(s^t)[a^i(s^t) + D^i_\lambda(s^t)] = 0$$

- We may not have $p(s^t) = \beta^t \pi(s^t) u'(c^i(s^t))/u'(c^i(s^0))$

Interpretation

Reputation Debt

- BR interpret $\mathrm{PV}(\lambda y^i|s^t)$ as the amount of debt merely due to the output drop after default
- In the terminology of BR, debt can be sustained, but only on the basis of the output drop
- Equivalently, there is no reputation debt

Extension to GE

- HL have proved that the First BR Theorem does not extend to GE
- We prove that the Second BR Theorem does extend to GE

Discussion

- Whatever small is λ, the bubble component of debt limits vanish
- The positive result in HL is not robust to a more realistic modeling of the consequences of default
- A relatively small change with respect to the default punishment produces a drastically different result
- There is a discontinuity when $\lambda \to 0$

Question

- Why should we interpret the difference $D^i_\lambda(s^t) - \mathrm{PV}(\lambda y^i|s^t)$ as the debt level due to the threat of credit exclusion (reputation debt)?
- Is it true that $\mathrm{PV}(\lambda y^i|s^t)$ is the debt level only due to the output drop (output drop debt)?

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- Is it true that $\mathrm{PV}(\lambda y^i|s^t)$ is the debt level only due to the output drop (output drop debt)?

The Example in HL: $\lambda = 0$

- Simple example with two agents
- In each period, one agent receives the high endowment and the other receives the low endowment
- They construct a symmetric Markov equilibrium

$$(q, (c^i, a^i, D_0^i)_{i \in I})$$

  with positive debt limits and interest rates lower than each agent's growth rates (zero risk-free interest rates)
- This economy is denoted by $\mathcal{E}^{\mathrm{HL}}$

The Example in HL with $\lambda > 0$

- For every $\lambda > 0$ we consider the economy $\mathcal{E}_\lambda^{\mathrm{HL}}$ where default entails credit exclusion and output drop
- We construct a symmetric Markov equilibrium

$$(q_\lambda, (c_\lambda^i, a_\lambda^i, D_\lambda^i)_{i \in I})$$

  with positive debt limits and interest rates higher than each agent's growth rates
- We show that

$$(q_\lambda, (c_\lambda^i, a_\lambda^i)_{i \in I}) \xrightarrow[\lambda \to 0]{} (q, (c^i, a^i)_{i \in I})$$

$$D_\lambda^i(s^t) = \lambda \, \mathrm{PV}(y^i | s^t) \xrightarrow[\lambda \to 0]{} D_0^i(s^t) > 0$$

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- For every $\lambda > 0$ we consider the economy $\mathcal{E}_\lambda^{\mathrm{HL}}$ where default entails credit exclusion and output drop
- We construct a symmetric Markov equilibrium

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  with positive debt limits and interest rates higher than each agent's growth rates
- We show that

$$(q_\lambda, (c_\lambda^i, a_\lambda^i)_{i\in I}) \xrightarrow[\lambda\to 0]{} (q, (c^i, a^i)_{i\in I})$$

$$D_\lambda^i(s^t) = \lambda\, \mathrm{PV}(y^i|s^t) \xrightarrow[\lambda\to 0]{} D_0^i(s^t) > 0$$

What does $\mathrm{PV}(\lambda y^i|s^t)$ represent?

- It does not seem reasonable to consider that $\mathrm{PV}(\lambda y^i|s^t)$ represents the debt sustained only on the basis of the output drop
- Indeed, we should have

$$\mathrm{PV}(\lambda y^i|s^t) \xrightarrow[\lambda \to 0]{} 0$$

since

$$J^i_\lambda(0,0|s^t) \xrightarrow[\lambda \to 0]{} J^i_0(0,0|s^t)$$

- $\mathrm{PV}(\lambda y^i|s^t)$ represents the current consumption agent i's is willing to give up in order to prevent the output drop when there is full commitment

$$J^i(N^i, -\mathrm{PV}(\lambda y^i|s^t)|s^t) = J^i_\lambda(N^i, 0|s^t)$$

Reputation Debt and High Interest Rates

- Since

$$D^i_\lambda(s^t) = \lambda\,\mathrm{PV}(y^i|s^t) \xrightarrow[\lambda\to 0]{} D^i_0(s^t) > 0$$

- A fraction of $D^i_\lambda(s^t)$ must reflect the utility loss due to the exclusion from credit markets
- In other words, some level of reputation debt must be sustained in the economy $\mathcal{E}^{\mathrm{HL}}_\lambda$ with high interest rates
- We propose an alternative way to disentangle
  - output drop debt Δ^i_λ
  - reputation debt R^i_λ

$$D^i_\lambda(s^t) = \Delta^i_\lambda(s) + R^i_\lambda(s^t)$$

Disentangling Repayment Incentives

- We suggest the following definition of output drop debt Δ_λ

$$J^i(D_\lambda, -\Delta_\lambda(s^t)|s^t) = J^i_\lambda(D^i_\lambda, 0|s^t)$$

- After default, the country looses λy^i but keeps the same access to credit markets

Disentangling Repayment Incentives

Proposition

$$\lambda y^i(s^t) < \Delta_\lambda(s^t) \leqslant \mathrm{PV}(\lambda y^i|s^t)$$

- Observe that

$$J^i(D^i_\lambda, -D^i_\lambda(s^t)|s^t) = J^i_\lambda(0,0|s^t) \leqslant J^i_\lambda(D^i_\lambda, 0|s^t) = J^i_\lambda(D^i_\lambda, -\Delta^i_\lambda(s^t)|s^t)$$

- Therefore $\Delta^i_\lambda(s^t) = \mathrm{PV}(\lambda y^i|s^t)$ if, and only if, after default the borrower does not benefit from keeping access to credit markets

An Important Property

Proposition

When the output drop parameter vanishes, the output drop debt also vanishes

$$\lim_{\lambda \to 0} \Delta_\lambda^i(s^t) = 0$$

- This result is expected since

$$J_\lambda^i(0, 0|s^t) \xrightarrow[\lambda \to 0]{} J_0^i(0, 0|s^t)$$

Positive Levels of Reputation Debt

- In our modification of the example in HL, we have

$$D^i_\lambda(s^t) \xrightarrow[\lambda \to 0]{} D^i_0(s^t) > 0$$

- This implies that

$$R^i_\lambda(s^t) := D^i_\lambda(s^t) - \Delta^i_\lambda(s^t) \xrightarrow[\lambda \to 0]{} D^i_0(s^t) > 0$$

- The level of reputation debt $R^i_\lambda(s^t)$ is positive even when $\lambda > 0$

Conclusion

- We show that part the debt level $\mathrm{PV}(\lambda y^i | s^t)$ is due to the threat of credit exclusion
- Our result is in sharp contrast with Bulow and Rogoff (1989)
- The reputation debt levels sustained through bubbles in HL can be approximated by reputation debt levels sustained under high interest rates (and small enough output costs)
- This is can be seen as a robustness property of the debt sustainability results in Hellwig and Lorenzoni (2008)

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An Example

- A deterministic economy with three countries $I = \{\mathrm{P}, \mathrm{R}_1, \mathrm{R}_2\}$
- The first country is "poor": its endowment is steeply decreasing to zero
  - growth rates are negative
  - the country is constantly in depression
- The other two countries are "richer" (zero growth)
  - Their endowments alternate between a high and a low value from one period to the next

Endowments of Rich Countries

- Identical Bernoulli functions u and choose $\overline{c} > \underline{c} > \delta$ such that

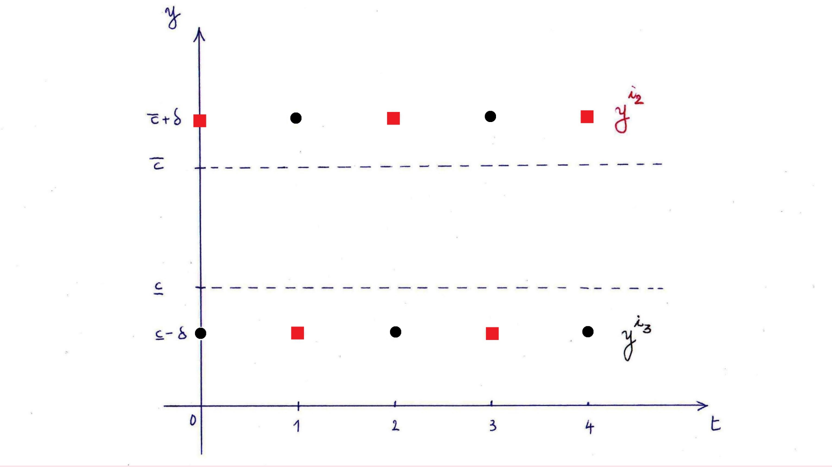
$$\beta u'(\underline{c}) = u'(\overline{c})$$

- Endowments of the rich countries

$$y_t^{\mathrm{R}1} = \begin{cases} \overline{c} + \delta & \text{if } t \text{ is even} \\ \\ \underline{c} - \delta & \text{if } t \text{ is odd} \end{cases} \quad \text{and} \quad y_t^{\mathrm{R}2} = \begin{cases} \underline{c} - \delta & \text{if } t \text{ is even} \\ \\ \overline{c} + \delta & \text{if } t \text{ is odd} \end{cases}$$

- There are gains to trade between the rich countries

Endowments of Rich Countries



$$\overline{c} > \underline{c} > \delta \quad \text{and} \quad \beta u'(\underline{c}) = u'(\overline{c})$$

Endowments of the Poor Country

- $y_0^{\mathrm{P}} = \overline{c} - \delta$
- $y_1^{\mathrm{P}} = \underline{c}$
- $(y_t^{\mathrm{P}})_{t \geqslant 2}$ is a decreasing sequence such that

$$\beta u'(y_t^{\mathrm{P}}) = u'(y_{t-1}^{\mathrm{P}})$$

- choose u such that[1]

$$\sum_{t \geqslant 2} y_t^{\mathrm{P}} < \infty$$

[1] Take for instance $u(c) = \ln(c)$ for $c \leqslant \overline{c}$

Sustaining Reputation Debt

- Choose $q_t = 1$ for every $t \geqslant 1$ (zero interest rates)
- Consider the following debt limits:

$$D_t^i = \begin{cases} \delta & \text{if } i = \mathrm{P} \\ 0 & \text{if } i = \mathrm{R}_1 \\ 0 & \text{if } i = \mathrm{R}_2 \end{cases}$$

- Debt limits satisfy exact roll-over, and are therefore ntt
- There is a competitive equilibrium where the poor country borrows

Details

Intermediation and Lending Incentives

- The poor country sustains positive levels of debt although interest rates are higher than its growth rates
  - The same repayment incentives as in BR
  - Self-enforcing debt limits are looser than the natural debt limits
- Rich countries are not creditworthy
- The poor country turns out to have a good reputation as a credible borrower
- The good reputation stems from its intermediation role helping rich countries to smooth consumption

Intermediation and Lending Incentives

- The poor country repays more than "its natural ability to repay"
- Why rich countries accept to lend "at infinite"

$$p_t(a_t^{\mathrm{R}1} + a_t^{\mathrm{R}2}) = \delta > 0$$

- This is because they are credit constrained and they need the poor country to act as a pass-through intermediary
- The poor country extracts the surplus δ for its financial services
- Having two potential lenders for which interest rates are lower than their growth rates is essential
- Otherwise, lending "at infinite" is not compatible with the transversality condition

Indeterminacy of debt constraints: Real effects

- Reputation can be split between countries i_1 and i_2
- When interest rates are lower than someone's endowment growth rates
- There is indeterminacy of creditworthiness or good reputation
- This indeterminacy has real effects

▸ Real Indeterminacy

The Need for Low Interest Rates

- Reputation debt must be a bubble
- A country's reputation debt can be positive even if interest rates are higher than its growth rates
- However, interest rates must be higher than someone's growth rates (not necessarily the borrower)

Proposition

Assume that $\lambda = 0$, if interest rates are higher than the every agent's growth rates, then there is no reputation debt

- Proof: the market transversality condition

Appendix

Equilibrium allocations

- Poor country P

$$c_t^{\mathrm{P}} = \begin{cases} \overline{c} & \text{if } t = 0 \\ y_t^{\mathrm{P}} & \text{if } t \geqslant 1 \end{cases} \quad \text{and} \quad a_t^{\mathrm{P}} = -\delta \quad \text{for } t \geqslant 1$$

- Rich country R_1

$$c_t^{\mathrm{R}1} = \begin{cases} \overline{c} & \text{if } t \text{ is even} \\ \underline{c} & \text{if } t \text{ is odd} \end{cases} \quad \text{and} \quad a_t^{\mathrm{R}1} = \begin{cases} 0 & \text{if } t \text{ is even} \\ \delta & \text{if } t \text{ is odd} \end{cases}$$

- Country R_2

$$c_t^{\mathrm{R}2} = \begin{cases} \overline{c} - \delta & \text{if } t = 0 \\ \overline{c} & \text{if } t = 1, 3, \dots \\ \underline{c} & \text{if } t = 2, 4, \dots \end{cases} \quad \text{and} \quad a_t^{\mathrm{R}2} = \begin{cases} \delta & \text{if } t \text{ is even} \\ 0 & \text{if } t \text{ is odd} \end{cases}$$

Return

Proof for the deterministic case ($x(s^t) \rightsquigarrow x_t$)

- Let (c, a) be an optimal plan under ntt debt limits
- The flow budget constraints are

$$p_t c_t + p_{t+1} a_{t+1} = p_t y_t + p_t a_t$$

- Observe that $a_t \geqslant -D_t \geqslant -\mathrm{PV}_t(y)$
- Since interest rates are lower than growth rates, there exists τ such that

$$p_\tau a_\tau = \min_{t \geqslant 1} p_t a_t$$

- The country defaults at time τ

The country defaults at time τ

- At time τ

$$p_\tau c_\tau + \underbrace{p_{\tau+1} a_{\tau+1} - p_\tau a_\tau}_{\geqslant 0} = p_\tau y_\tau$$

- At any time $t > \tau$

$$p_t c_t + \underbrace{p_{t+1} a_{t+1} - p_\tau a_\tau}_{\geqslant 0} = p_t y_t + \underbrace{p_t a_t - p_\tau a_\tau}_{\geqslant 0}$$

- Let $\tilde{a}$ be defined by

$$p_{t+1} \tilde{a}_{t+1} = p_{t+1} a_{t+1} - p_\tau a_\tau$$

- $\tilde{a}$ finances consumption c without the need for borrowing
- Actually, we should choose the largest τ satisfying

$$p_\tau a_\tau = \min_{t \geqslant 1} p_t a_t$$

Return

Real Indeterminacy

- With the same zero interest rates, there is a continuum of equilibria with ntt debt limits
- The gains to trade between the rich countries can be partially intermediated by the poor country
- For any $\alpha \in [0, 1]$, we can exhibit an equilibrium where

$$D_t^i = \begin{cases} \alpha\delta & \text{if } i = \mathrm{P} \\ 0 & \text{if } i = \mathrm{R}_1 \\ (1-\alpha)\delta & \text{if } i = \mathrm{R}_2 \end{cases}$$

Return

Proof

- There exists (Fixed-point Theorem) a non-negative process $\underline{D}^i$ such that

$$\underline{D}^i(s^t) = \lambda y^i(s^t) + \sum_{s^{t+1} \succ s^t} q(s^{t+1}) \min\{D^i_\lambda(s^{t+1}), \underline{D}^i(s^{t+1})\}$$

- We can show that

$$J^i(D_\lambda, -\underline{D}^i(s^t)|s^t) \geqslant J^i_\lambda(0, 0|s^t)$$

which implies that $\underline{D}^i(s^t) \leqslant D^i_\lambda(s^t)$

- We then get that

$$\underline{D}^i(s^t) = \lambda y^i(s^t) + \sum_{s^{t+1} \succ s^t} q(s^{t+1}) \underline{D}^i(s^{t+1})$$

Return

Proof

- Let $(c, a) \in d(D_\lambda, -D_\lambda(s^t)|s^t)$
- Denote by $\widetilde{\mathcal{E}}$ the economy where y is replaced by $\tilde{y} := (1 - \lambda)y$
- Let $\tilde{a} := a + \mathrm{PV}(\lambda y)$ and $\tilde{D}_0 := D_\lambda - \mathrm{PV}(\lambda y)$ then

$$(c, \tilde{a}) \in \tilde{d}(\tilde{D}_0, -\tilde{D}_0(s^t)|s^t)$$

  and

$$\tilde{J}(\tilde{D}_0, -\tilde{D}_0(s^t)|s^t) = J(D_\lambda, -D_\lambda(s^t)|s^t) = J_\lambda(0, 0|s^t) = \tilde{J}(0, 0|s^t)$$

- Applying the characterization result in HL, we get that $\tilde{D}_0 = M$ where M satisfies exact roll-over (bubble)

Return